

# Dynamic Master selection in wireless networks

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**Abstract.** Mobile wireless networks need to maximize their network lifetime (defined as the time until the first node runs out of energy). In the broadcast network lifetime problem, all nodes are sending broadcast traffic, and one asks for an assignment of transmit powers to nodes, and for sets of relay nodes so that the network lifetime is maximized. The selection of a dynamic relay set consisting of a single node (the ‘master’), can be regarded as a special case, providing lower bounds to the optimal lifetime in the general setting. This paper provides a first analysis of a ‘dynamic master selection’ algorithm.

## 1 Introduction

Mobile wireless networks are often battery powered which makes it important to maximize the network lifetime. Here, the network lifetime is defined as the time until the first node runs out of energy. The broadcast network lifetime problem asks for settings of transmit powers and (node-dependent) sets of relay nodes, that maximize the network lifetime, while all nodes originate broadcast traffic. Literature in this area considers the lifetime maximization in mobile ad-hoc networks (MANETs). Often, the complexity is reduced by assuming transmissions originate from a single source ([3], [5] and [7]). The related problem of minimizing the total energy consumption for broadcast traffic has also been widely studied, because it provides a crude upper bound to the lifetime of the network. In [4] and [1] it is shown that minimizing the total transmit power is NP-hard. Another way to reduce the complexity is to allow transmissions from multiple sources but ask for a node independent *set* of relay nodes to maximize the network lifetime. This leads to lower bounds for the general network lifetime problem. This paper presents a first analysis of a special case, where we ask for a *single* relay node (the master), which is allowed to change over time.

## 2 General model and notation

We assume all nodes can reach each other when transmitting at maximum power. For a set  $V \subseteq \mathbb{R}^d$  of potential master nodes, a power assignment is a function  $p : V \rightarrow \mathbb{R}$ . To each ordered pair  $(u, v)$  of transceivers we assign a transmit power threshold, denoted by  $c(u, v)$ , with the following meaning: a signal transmitted by transceiver  $u$  can be received by  $v$  only when the transmit power is at least  $c(u, v)$ . We assume that  $c(u, v)$  are known, and that these are symmetric. For a node  $m \in V$ , let  $p_m$  denote the power assignment  $p_m : V \rightarrow \mathbb{R}$  defined as:

$$p_m(v) = \begin{cases} c(v, m) & \text{for } v \neq m, \\ \max_{v \in V} c(v, m) & \text{for } v = m. \end{cases} \quad (1)$$

Each vertex is equipped with battery supply  $b_v$ , which is reduced by amount  $\lambda p_m(v)$  for each message transmission by  $v$  with transmit power  $p_m(v)$ . Similarly,  $b_v$  is reduced by amount  $\mu r(v)$  for each reception. Let  $T_1, T_2, T_3, \dots$  denote the time periods. Let node  $i$  transmit  $a_i(T_j)$  times during time period  $T_j$ . We assume that the  $a_i(T)$  are constant for all  $T_i$ , ( $i = 1, \dots$ ), and define  $a_i = a_i(T)$ . We call a series of transmissions where each node  $i$  transmits  $a_i$  times a *round*. Suppose node  $m$  is master. With these assumptions, we obtain after one round:

$$b_v = \begin{cases} b_m - \lambda p_m(m) \sum_{v \in V} a_v - \mu r(m) \sum_{v \neq m} a_v & \text{for } v = m, \\ b_v - \lambda a_v p_m(v) - \mu r(v) \sum_{v \in V} a_v & \text{for } v \neq m. \end{cases}$$

In [2] we analyzed the case where a master  $m$  is chosen which is kept for the whole lifetime of the network. This paper is concerned with the following problem: given a graph  $G = (V, E, c, b, a)$ ,  $c : E \rightarrow \mathbb{R}$  denotes the transmit power thresholds, and  $b : V \rightarrow \mathbb{R}$  denotes the initial battery levels  $b_v, v \in V$ , and the relative frequencies  $a_1, \dots, a_n$ , one asks for times  $x_v \geq 0$  for each node  $v$  to be master in such a way that  $L(G, x) = \sum_{v \in V} x_v$  is maximized under the condition that the remaining battery capacity of each node is positive during the lifetime of the network. In this paper, we assume  $\lambda = 1$  (by scaling),  $V \subseteq \mathbb{R}^d$ ,  $E$  corresponds to a complete graph,  $c(u, v) = \|u - v\|^2$ . We also assume  $\mu = 0$ , which is consistent with many long-range radio systems, where transmit power dominates the signal processing power.<sup>1</sup> We call  $x = (x_1, \dots, x_n) \in \mathbb{N}_+^n$  *feasible* if for all  $m \in V$ ,

$$b_m - \lambda \sum_{v \neq m} a_v x_v p_v(m) - \lambda x_m p_m(m) \sum_{v \in V} a_v \geq 0. \quad (2)$$

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<sup>1</sup> The analysis presented above is straightforwardly extendable to the case  $\mu \neq 0$ .

The terms  $\lambda \sum_{v \neq m} a_v p_v(m)$  and  $\lambda x_m p_m(m) \sum_{v \in V} a_v$  in (2) indicate the reduction in battery capacity of node  $m$  during the periods when nodes  $v \neq m$  are master, and when  $m$  is master, respectively.

Now (2) can be rephrased as:  $Ax \leq b$ , where  $b : V \rightarrow \mathbb{R}^+$ , and where  $A$  is an  $n \times n$ -matrix where the entry corresponding to  $(v, m)$  is defined by:

$$A(v, m) = \begin{cases} p_m(m) \sum_{v \in V} a_v & \text{for } v = m, \\ a_v p_v(m) & \text{for } v \neq m. \end{cases} \quad (3)$$

In Section 3 of this paper we compare dynamic master selection algorithms for the continuous power case. In Section 4 we address the impact of supporting only a discrete set of transmit power levels. Section 5 presents the conclusions.

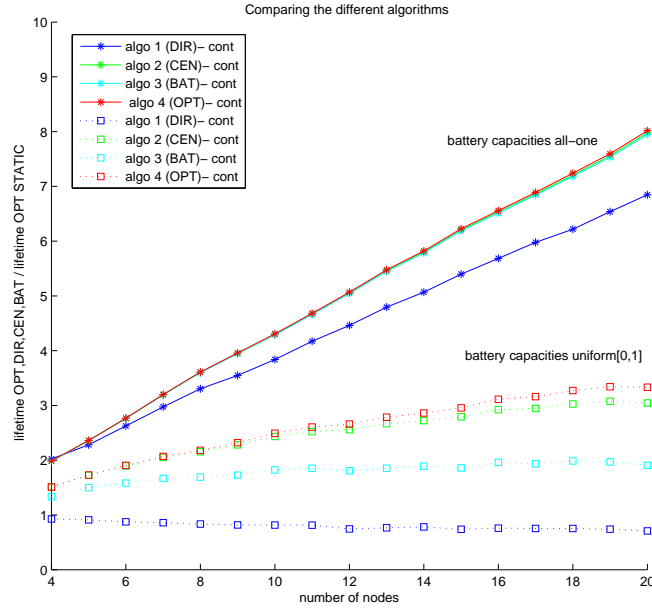
### 3 The continuous power case

The network lifetime in number of rounds was evaluated for  $n$ , ranging from 4 to 20. The nodes were uniformly distributed in a two dimensional disk of unit diameter. For each algorithm, the average network lifetime was evaluated over 1000 simulations. The relative message transmission frequencies were  $a_v = 1$  for  $v \in V$ . The following algorithms were compared:

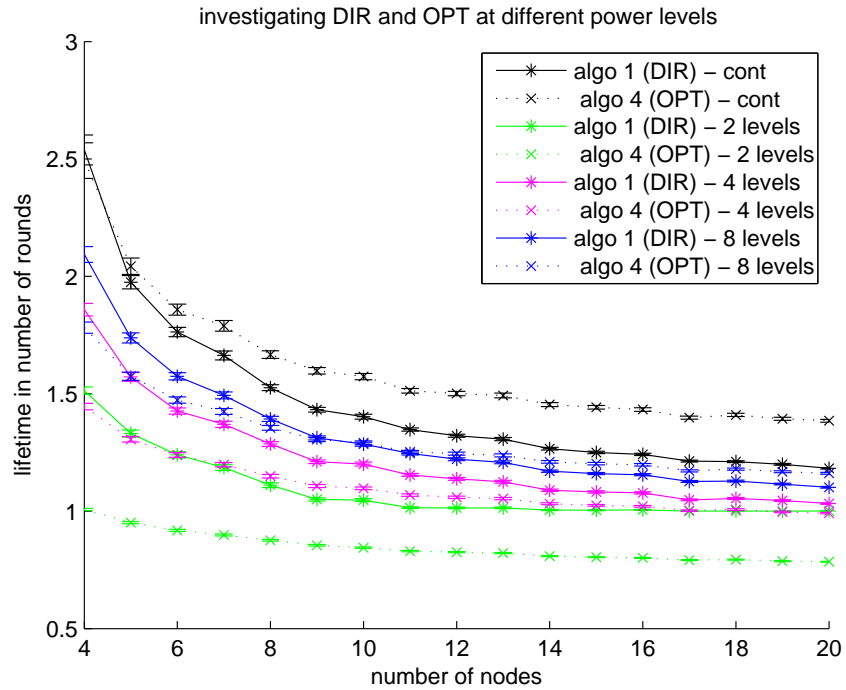
- *Optimal Master Selection (OPT)*. Choose  $x \geq 0$ , so that  $L(G, x)$  is maximized, under condition (2).
- *Central Master Selection (CEN)*. Choose  $x$ , by periodically selecting performing the optimal static master node selection.
- *Maximum Battery Master Selection (BAT)*. Choose  $x$  by periodically selecting a master node in such a way that (at the update time  $t$ )  $b_m$  is maximal among  $b_v$  for  $v \in V$ .
- *Direct Transmission (DIR)*. There is no master: all nodes reach all other nodes via a single hop transmission.

We include it for reference purposes.

In Figure 1(a), we compare the ratio of lifetime for the algorithm to the lifetime of the optimal static algorithm (as in [2]). Two cases are displayed: all-one battery capacities:  $b_v = 1$  for all  $v \in V$ , and  $b_v \cong U(0, 1), v \in V$ . The simulations show that dynamic master selection extends the lifetime significantly compared to static master selection. In order of decreasing lifetime the algorithms are : OPT, CEN, BAT and DIR. OPT and CEN



(a) Simulation results for the continuous power case with battery capacities all-one and uniformly distributed.



(b) Comparing DIR and OPT for continuous and 2 and 8 discrete power case with all-one battery capacities.

**Fig. 1.** Simulation results for dynamic master selection.

are close, and we expect that CEN and OPT are equal when considering infinitesimal time periods. The improvement depends strongly on the initial battery capacities: for uniformly  $[0,1]$  battery capacities this factor is about 3 (for 15 nodes or more), for the all-one battery capacities -where the total amount of energy in the network is, on average, doubled- this factor amounts to at least 6. In this case OPT, CEN and BAT are very close. For the case of uniform  $[0,1]$  battery capacities even *static* master selection is better for the network lifetime than direct routing (shown by the blue squared dotted line dropping below one for increasing number of nodes). As the dynamic master selection is a highly specific case of ad-hoc multihop routing, this indicates that introducing multihop routing functionality is beneficial for the network lifetime, provided the transmit power levels are continuously adjustable. Work is in progress to support these simulation results with mathematical analysis.

#### 4 Restricting the number of power levels

In practice, often only a discrete set of transmit power levels is supported in hardware and software. In the extreme case only one constant power level is supported. In contrast to the previous section it is immediately clear that in the constant power case DIR outperforms multihop routing, due to the fact that multihop routing reduces the battery by a constant at each transmission for (at least) 2 nodes. In Figure 1(b) we investigate how many power levels need to be supported before OPT outperforms DIR. Simulations with  $U[0,1]$ -distributed battery capacities (not displayed) show OPT outperforms DIR already for 2 power levels. However, the figure shows that, with all-one battery capacities, 2 power levels is not enough. For 8 power levels OPT outperforms DIR for 10 nodes or more. However, with 4 or less power levels, DIR outperforms OPT.

As a special case of the fixed number of power levels, we address the constant power case. Here, the matrix  $A$  as defined in (3) equals  $A = (n-1)pI_n + pE_n$ , where  $I_n$  denotes the identity matrix and  $E_n$  the all-one matrix. Clearly direct transmission leads to a lifetime, in rounds  $L = \min\{b_i/p\}$ . For the OPT we obtain:

**Theorem 1** *Let  $G = (V, c, b)$  be given, and  $n \geq 2$ . Then the network lifetime for algorithm OPT is*

$$L(G) = \min_{v \in V} \left\{ b_v, \frac{\sum_{v \in V} b_v}{p(2n-1)} \right\} \quad (4)$$

*Proof.* W.l.o.g.  $V = \{1, \dots, n\}$ ,  $p = 1$  and  $b_1 \leq \dots \leq b_n$ . By LP duality  $\max\{1^T x | Ax \leq b, x \geq 0\} = \min\{y^T b, yA \geq 1, y \geq 0\}$ , where  $y^T$  denotes the transpose of a vector, and 1 denotes the all-one vector. Considering  $y = (2n - 1)^{-1} 1^T$ , it follows that  $\sum x_i \leq (2n - 1)^{-1} \sum_{v \in V} b_v$ . To see the other upper bound, consider  $y = [1, 0, \dots, 0]$ , which implies that  $nx_1 + \sum_{i=2}^n x_i \leq b_1$ , whence also  $\sum_{v \in V} x_v \leq b_1$ . To see that the upper bounds are attainable, first assume  $b_1 \geq \sum_{i=1}^n b_i / (2n - 1)$ . Next consider  $x$  as given by  $x_i = (b_i - \frac{\sum b_i}{2n - 1}) / (n - 1)$ . By assumption  $x$  is feasible. Moreover:  $\sum x_i = \sum b_i / (2n - 1)$  by simple substitution. To see that the lower bound  $b_1$  is attainable, assume ((2)) does not hold, so  $b_1 < \sum_{i=1}^n b_i / (2n - 1)$ . Choose  $x_1 = 0$ , and repeat this procedure until we are back in the situation under (a). With the corresponding assignment also the lifetime  $b_1$  is realized.

## 5 Conclusions and future work

When the transmit power can be regarded as a *continuous variable*, we find that dynamic master selection algorithms extend the network lifetime significantly compared to static master selection. In order of decreasing lifetime the algorithms are : OPT, CEN, BAT and DIR. The improvement depends strongly on the initial battery capacities. Work is in progress to support these simulation results with mathematical analysis as in [2]. For *discrete power levels*, dynamic master selection can only improve upon direct routing, when there are at least two power levels. Our results suggest that 8 power levels are sufficient for multihop routing to have longer network lifetime than direct transmission, except for small networks.

## References

1. Cagalj, M., Hubaux, J., Enz, C.: Minimum-energy broadcast in all-wireless networks, NP-completeness and distribution issues. In: Proceedings of the Annual International Conference on Mobile Computing and Networking, MOBICOM, pp 172–182 (2002)
2. Maurits de Graaf, Jan-Kees van Ommeren: Increasing network lifetime by battery-aware master selection in radio networks, Proceedings of the 3rd ERCIM workshop on eMobility, May 2009, Netherlands, pp. 3-14.
3. Kang, I., Poovendran, R.: Maximizing Network Lifetime of Broadcasting over Wireless Stationary Ad Hoc networks, Mobile Networks and Applications, 10, 879–89, (2005)
4. Liang, W.: Constructing minimum-energy broadcast trees in wireless ad hoc networks, In: Proceedings of the International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc), pp. 112–122 (2002)
5. Pow, C.P., Goh, L.W.: On the construction of energy-efficient maximum residual battery capacity broadcast trees in static ad-hoc wireless networks, Computer Communications, 29, 93–103 (2005)

6. Lloyd, E., Liu, R., Marathe, M., Ramanathan, R., Ravi, S.: Algorithmic Aspects of Topology Control problems for ad-hoc networks, *Mobile Networks and applications*, 10, Issue 1-2 , 19–34 (2005)
7. Park, J., Sahni, S.: Maximum lifetime broadcasting in wireless networks. In: 3rd ACS/IEEE International Conference on Computer Systems and Applications, 2005:1-8 (2005)